Transonic Flow Past a Backward-Facing Step

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EARLY studies of flow past a backward-facing step within the supersonic flow regime,[1,2] were useful to illustrate and identify the significance of the viscous flow mechanisms in conjunction with their interaction with the outer free stream. The analysis of these problems within the transonic flow regime, however, has been greatly hampered by lack of an adequate and efficient method to establish the inviscid flowfield and lack of a model to account for the aforementioned viscid-inviscid interaction whose outer inviscid flow is governed by elliptic differential equations. Since the suggestion of dealing with the inviscid transonic flow through numerical treatment was made by Murman and Cole,[3] considerable progress[4-7] has been made in this area. Inviscid transonic axisymmetric flowfield can now be established through numerical calculations of the full potential equation with a reasonable amount of computer time even when the corresponding viscous flow modification to the inviscid body geometry is not negligible.[6,7] It is the intention of this Note to present a flow model to account for the viscid-inviscid interaction within this transonic flow regime and to report the results obtained from such a study. Moreover, it is hoped that this discussion will promote and stimulate additional interest, especially in the experimental aspect of the problem, so that proper guidance can be provided toward the solution of these outstanding separated flow problems within the transonic flow regime.

From the experience gained in the study of separated flows within the incompressible flow regime,[8] it is suggested that the viscid-inviscid interaction can be properly accounted for through the establishment of a corresponding inviscid flowfield. For a flow past a backward-facing step, this corresponding inviscid flowfield is interpreted as if it were generated from a flow past an equivalent body whose geometry is given in dimensionless form by (see Fig. 1)

$$\begin{aligned} y_b &= SH & x &\le 0 \\ y_b &= SH(1-x^3) & 0 &\le x \le 1 \\ y_b &= 0 & x &\ge 1 \end{aligned} \tag{1}$$

where point (1,0) is the inviscid rear stagnation point.‡ For a given parametric value of the shape factor SH, the inviscid transonic flowfield associated with such a body may be established through detailed numerical calculations. Description of such detailed calculations, as well as the argument leading to the selection of such an equivalent body geometry, can be found in Ref. 9. One set of these typical results of calculations is presented in Fig. 2.

With the established inviscid flowfield, viscous flow processes (e.g., turbulent boundary-layer buildup along the upper wall, turbulent mixing along the wake, recompression and subsequent reattachment) may be calculated within the framework of the boundary-layer concept. The spread of the viscous region would automatically identify the state of the fluid at the edge of the viscous layer from the already established inviscid flowfield. The basic concept of viscid-inviscid interaction will be properly manifested if these viscous flow processes can serve to determine the correct value of the shape factor SH needed to establish the corresponding inviscid flow.

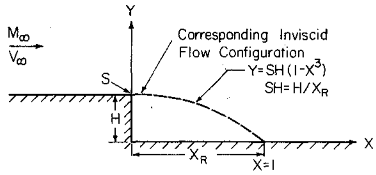

Fig. 1 Transonic flow past a backstep—illustrating also the corresponding inviscid flow configuration.

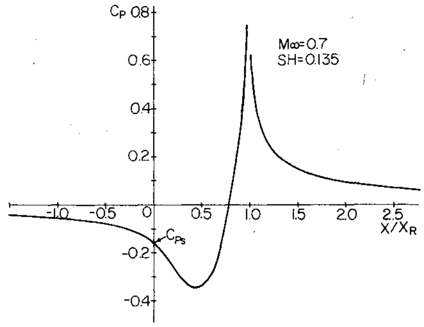

Fig. 2 Surface pressure coefficient of equivalent body.

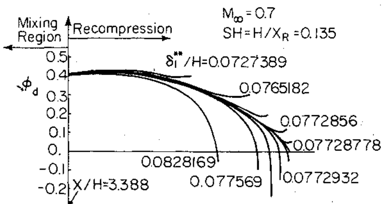

Fig. 3 Saddle point behavior of the point of reattachment.

The analyses of mixing, recompression, and reattachment follow essentially the same integral approach as discussed for other flow regimes[8,10,11] and is reported in detail in Ref. 9. Indeed, calculations of the viscous flow processes have shown that the point of reattachment behaves as a saddle point singularity for the system of equations describing the viscous flow process during recompression, which, in turn, leads to the establishment of a set of compatible parametric values for the problem within this transonic flow regime. This is further explained as follows.

For a specific Mach number of approaching flow, such calculations would determine the flowfield corresponding to a specific boundary-layer thickness at the step (or a specific characteristic Reynolds number). It has been found that to achieve such a purpose, many sets of inviscid flow calculations with different SH values must be performed and the computational cost in producing these results is prohibitively high. Instead, one is forced to determine the correct initial momentum thickness ratio of the boundary layer of the flow at the step compatible with the inviscid flowfield with a specific value of the shape parameter SH. A typical set of results obtained from calculations for the viscous flow recompression is shown in Fig. 3, where x_r represents the distance along recompression region. The saddle point character of the point of reattachment is fully illustrated. The dividing streamline velocity behaves in entirely different manners as the point of reattachment is approached for slightly different values of the initial momentum thickness ratio. Smaller values of the latter would eventually result in an increase of the former, whereas slightly larger latter values would reduce the former so drastically that the dividing streamline stagnates before the lower wall is reached.

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‡This inviscid rear stagnation point should not be confused with the point of reattachment of the real flow.

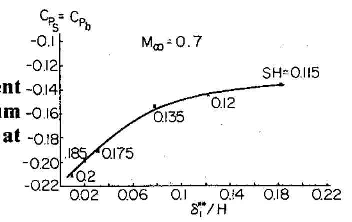

Fig. 4 Base-pressure coefficient as influenced by the momentum thickness of the boundary layer at the step ($M_\infty = 0.7$).

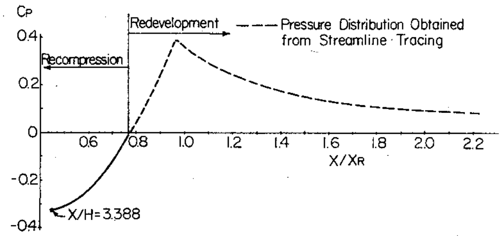

Fig. 5 Wall-pressure distribution within regions of recompression and redevelopment ($M_\infty = 0.7, SH = 0.135$).

Since only one parameter is to be determined from these calculations, it is not necessary to carry out calculations beyond the establishment of perhaps the third digit after the decimal point for the initial momentum thickness ratio if the pressure distribution on the lower wall is not of interest or concern.

If one implies from the boundary-layer concept that the pressure at the step is the base pressure, Fig. 4 presents the base-pressure coefficient obtained from this series of investigations. As shown in this figure, the base-pressure ratio indeed is dependent upon the Reynolds number. Although there are no available experimental data for comparison purposes, a solution for $c_p = -0.207$, obtained previously[8] for incompressible flow with $\delta_l^{**}/H = 0.0178$ which has been checked favorably against experimental investigations by Tani and Tanner indicates that these results are reasonable.

Figure 5 shows the established wall-pressure distribution up to the point of reattachment for a specific case of calculations. In addition, the pressure distribution is extended downstream of reattachment by tracing the streamline starting at the edge of the viscous layer at the section of reattachment. An overshoot of static pressure occurs before it decays asymptotically toward that of the free stream. The major difficulty in the calculation of flow redevelopment after reattachment lies in the nonequilibrium nature of the turbulence structure. The decay of turbulence due to the presence of wall in conjunction with the drastic rise of the wall shear stress that has been observed in incompressible flow cases should also prevail in the present situation. Experimental explorations are needed before additional progress can be made in this aspect of the problem.

It should be pointed out that, with the flowfield established thus far, the corresponding invisic body geometry will be different from what has been assumed from Eq. (1), which corresponds to the limiting case of infinite Reynolds number.§ The profile of the equivalent body for the established viscous flow with a finite Reynolds number should be at a distance of the "local displacement thickness" of the viscous layer from the dividing streamline, and it would not coincide with the lower wall downstream of the wake. Therefore, the results presented here can only be considered as a first approximation to the problem. Since the inviscid flow is established through finite difference calculations, adaptation of the corrected profile due to the effect of finite Reynolds number for improvement purposes is possible, although such a scheme would be extremely time consuming.¶ Nevertheless, it is expected that this scheme of successive approximation is a rapidly convergent process since it is well known that the effect of Reynolds number at high Reynolds number regime is very small. Perturbation of the equivalent body profile for separated flows seems to be a useful concept.

For higher approaching subsonic flow Mach numbers, the pressure at the step may approach that of the free stream even though the base-pressure ratio may be reduced. This is obviously true when the approaching flow is sonic ($M = 1$). Under this situation, the Mach number at the step is also sonic, whereas the base is impressed by a pressure resulting from a Prandtl-Meyer expansion. Although the equivalent body should exhibit a slope discontinuity at the step to simulate the Prandtl-Meyer expansion, it is doubtful whether finite difference calculations can reproduce such an expansion at the corner. Fortunately, the method of characteristics is available under this situation although insertion of such an equivalent body with a slope discontinuity remains to be an interesting alternative.

It should be noted that under assumptions adopted for the steady viscous flow recompression, the point of reattachment exhibits itself as a saddle point singularity for the system of equations governing the flow and this special character can be exploited to good advantage. Although the physical process does not seem to present such sensitive characteristics, as associated with a saddle point, the validity of this mathematical behavior should be judged only by the merits of its ultimate results when compared with experimental data. Huxley once said that what one could get out of the mathematical mill depended solely on what one put into it. It is well known that, as long as one adopts the Navier-Stokes equation to describe the steady viscous fluid motion and simultaneously demands continuous solutions to the equation, saddle point singularity or singularities of other types may occur.

§For separated flows, the case of infinite characteristic Reynolds number is usually different with the case of inviscid flow.

¶It takes 15 min to establish one set of inviscid flow results with an IBM 360/75 computer system.

Acknowledgment

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